4-9 Solving Radical Inequalities Notes

When restricted to the real number system, what must be true about the radicand when taking the square root?

The radicand must be greater than zero (positive)

So, what would $x$ have to be in the following expressions?

\[
\begin{align*}
\sqrt{x-4} & \quad \sqrt{5x-8} & \quad \sqrt{3x+9} \\
x-4 & \geq 0 & \quad 5x-8 & \geq 0 & \quad 3x+9 & \geq 0 \\
x & \geq 4 & \quad 5x & \geq 8 & \quad 3x & \geq -9
\end{align*}
\]

When solving radical inequalities, you have to solve 2 parts.

1. **Solve the inequality**
2. **Solve for the radicand to be $\geq 0$**

Examples:

\(\sqrt{2x+4} \leq 4\)

**Step 1:**
\[
2x+4 \leq 16 \\
x \leq 6
\]

**Step 2:**
\[
2x+4 \geq 0 \\
x \geq -2
\]

**Solution:** \([-2, 6]\)

\(\sqrt{x-3} + 2 \leq 5\)

**Step 1:**
\[
(x-3) \leq (3)^2 \\
x-3 \leq 9 \\
x \leq 12
\]

**Step 2:**
\[
x-3 \geq 0 \\
x \geq 3
\]

**Solution:** \([3, 12]\)

\(\sqrt{8x+1} \geq 7\)

**Step 1:**
\[
(\sqrt{8x+1})^2 \geq (7)^2 \\
8x+1 \geq 49 \\
x \geq 6
\]

**Step 2:**
\[
8x+1 \geq 0 \\
x \geq -\frac{1}{8}
\]

**Solution:** \([-\frac{1}{8}, 6]\)

\(\sqrt{x-7} + 9 < 12\)

**Step 1:**
\[
(x-7)^2 \leq (3)^2 \\
x-7 \leq 9 \\
x \leq 16
\]

**Step 2:**
\[
x-7 \geq 0 \\
x \geq 7
\]

**Solution:** \([7, 16)\)
Algebra 2 – Unit 4 Square Root Functions

\[ \sqrt{x - 4} \leq (\sqrt{2x - 13})^2 \]

Step 1:
\[ \frac{x - 4}{2} \leq 2x - 13 \]
\[ x - 4 \leq 4x - 26 \]
\[ 9 \leq x \text{ or } x \geq 9 \]

Step 2:
\[ x - 4 \geq 0 \]
\[ x \geq 4 \]
\[ 2x - 13 \geq 0 \]
\[ 2x \geq 13 \]
\[ x \geq \frac{13}{2} \]

Solution:
\[ (9, \infty) \]

\[ 2 + \sqrt{2x + \sqrt{14}} \]

(beware careful!!)

Step 1:
\[ 2 + 2\sqrt{2x} + x \geq x + 14 \]

Step 2:
\[ 2\sqrt{2x} \geq 12 \]
\[ \sqrt{2x} \geq 6 \]
\[ (\sqrt{2x})^2 \geq (6)^2 \]
\[ 2x \geq 36 \]
\[ x \geq 18 \]

Solution:
\[ [18, \infty) \]

In order to compete in the America’s Cup sailboat race, a boat must satisfy the rule,

\[ l + 1.25\sqrt{s} - 9.8\sqrt{d} \leq 16 \]

where \( l \) is the length (in meters) of the boat, \( s \) is the area (in square meters) of the sails, and \( d \) is the volume (in cubic meters) of water displaced by the boat. A boat has a length of 20 meters and displaces 27 cubic meters of water. What is the maximum allowable value for \( s \)?

\[ l = 20 \]
\[ d = 27 \]

\[ 1.25\sqrt{s} - 9.4 \leq 16 \]

\[ 1.25\sqrt{s} \leq 25.4 \]

\[ \frac{1.25\sqrt{s}}{1.25} \leq \frac{25.4}{1.25} \]

\[ (\sqrt{s})^2 \leq (20.32)^2 \]

\[ 0 \leq s \leq 412.9 \text{ m}^2 \]